<u>8.1, 8.2, and 10.1 Worksheet</u>

1. Find the length of the arc made by the equation $y = x^{\frac{3}{2}}$ and bounded by the points (1,1) and (4,8).

$$L = \int_a^b \sqrt{1+(f'(x))^2}\, dx$$

$$f(x) = x^{3/2}, \quad f'(x) = \frac{3}{2}x^{1/2}$$

$$L = \int_1^4 \sqrt{1+\left(\frac{3}{2}x^{1/2}\right)^2}\, dx = \int_1^4 \sqrt{1+\frac{9}{4}x}\, dx$$

let $u = 1+\frac{9}{4}x, \quad du = \frac{9}{4}dx$

$$\frac{4}{9}\int_{x=1}^{x=4} \sqrt{u}\, du = \left[\frac{4}{9}\left(\frac{2}{3}u^{3/2}\right)\right]_{x=1}^{x=4}$$

$$\left[\frac{4}{9}\left(\frac{2}{3}\left(1+\frac{9}{4}x\right)^{3/2}\right)\right]_1^4$$

$$= \frac{8}{27}\left(1+\frac{9}{4}(4)\right)^{3/2} - \frac{8}{27}\left(1+\frac{9}{4}(1)\right)^{3/2}$$

$$= \boxed{\frac{8}{27}(10)^{3/2} - \frac{8}{27}\left(\frac{13}{4}\right)^{3/2}}$$

2. Find the surface area obtained by rotating the equation $y = \sqrt{4 - x^2}$ around the x-axis from [-1,1].

$$SA = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = \sqrt{4-x^2} = (4-x^2)^{1/2}, \quad \frac{dy}{dx} = \frac{1}{\cancel{2}}(4-x^2)^{-1/2} \cdot -\cancel{2}x$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$SA = \int_{-1}^{1} 2\pi \cdot \sqrt{4-x^2} \cdot \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= \int_{-1}^{1} 2\pi \cdot \sqrt{4-x^2} \cdot \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-1}^{1} 2\pi \cdot \sqrt{4-x^2} \cdot \sqrt{\frac{4-\cancel{x^2}}{4-x^2} + \frac{\cancel{x^2}}{4-x^2}}\, dx$$

$$= \int_{-1}^{1} 2\pi \cdot \sqrt{4-x^2} \cdot \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= \int_{-1}^{1} 2\pi \cdot \cancel{\sqrt{4-x^2}} \cdot \frac{2}{\cancel{\sqrt{4-x^2}}}\, dx$$

$$= \int_{-1}^{1} 2\pi \cdot 2\, dx = \int_{-1}^{1} 4\pi\, dx$$

$$= \left[4\pi x\right]_{-1}^{1} = 4\pi(1) - 4\pi(-1) = 4\pi + 4\pi = \boxed{8\pi}$$

3. Find the length of the arc $y = x^2$ bounded by the points (1,1) and (2,4). Then, find the surface area of the solid obtained by rotating this arc around the y-axis.

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad y = x^2, \ \frac{dy}{dx} = 2x$$

$$= \int_1^2 \sqrt{1 + (2x)^2}\, dx = \underbrace{\int_1^2 \sqrt{1+4x^2}}\, dx \approx \boxed{3.167841}$$

*can use trig sub!
just plugging into calculator
+ estimating here though

*because it's around y-axis, we have:

$$SA = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$y = x^2 \rightarrow x = \sqrt{y}, \ \frac{dx}{dy} = \frac{1}{2\sqrt{y}}$$

$$SA = 2\pi \int_1^4 \sqrt{y} \cdot \sqrt{1 + \left(\frac{1}{2\sqrt{y}}\right)^2}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y} \cdot \sqrt{1 + \frac{1}{4y}}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y + \frac{1}{4}}\, dy \qquad \text{let } u = y + \frac{1}{4}$$
$$du = 1\, dx$$

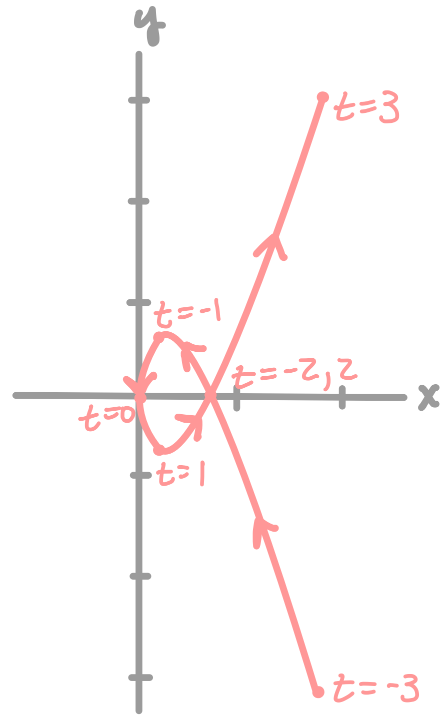
$$= 2\pi \int_{y=1}^{y=4} \sqrt{u}\, du$$

$$= 2\pi \left[\frac{2}{3} u^{3/2}\right]_{y=1}^{y=4} = \frac{4\pi}{3}\left[\left(y + \frac{1}{4}\right)^{3/2}\right]_1^4$$

$$= \boxed{\frac{4\pi}{3}\left[\left(\frac{17}{4}\right)^{3/2} - \left(\frac{5}{4}\right)^{3/2}\right]}$$

4. Sketch the curve by using the parametric equations, $x = t^2$, $y = t^3 - 4t$, $-3 \le t \le 3$, to plot points. Indicate with an arrow the direction in which the curve is traced as t increases. Eliminate the parameter.

t	$x=t^2$	$y=t^3-4t$
-3	9	-15
-2	4	0
-1	1	3
0	0	0
1	1	-3
2	4	0
3	9	15



$x = t^2,\ y = t^3 - 4t$

$t = \pm\sqrt{x} \rightarrow y = (\sqrt{x})^3 - 4(\sqrt{x}) \rightarrow \boxed{y = \sqrt{x^3} - 4\sqrt{x}}$

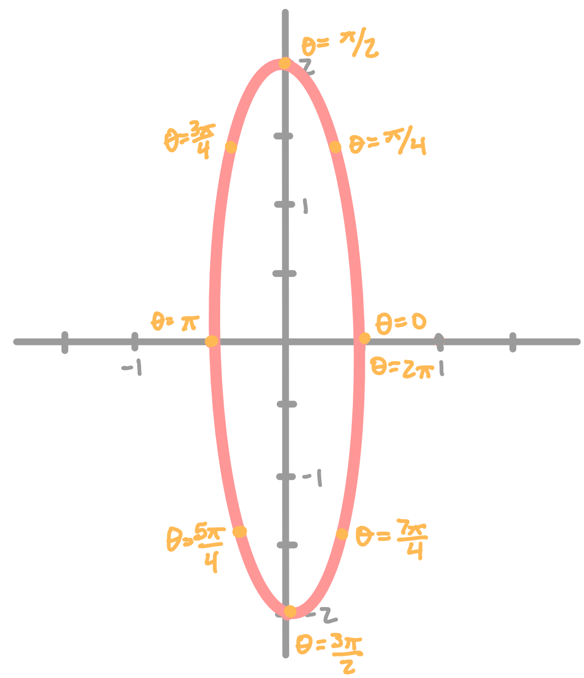
and

$y = (-\sqrt{x})^3 - 4(-\sqrt{x}) \rightarrow \boxed{y = 4\sqrt{x} - \sqrt{x^3}}$

5. Sketch the curve by using the parametric equations,

$x = \frac{1}{2}cos\theta,\ y = 2sin\theta,\ 0 \leq \theta \leq 2\pi$, to plot points. Indicate with an arrow the direction in which the curve is traced as t increases. Eliminate the parameter.

θ	$x=\frac{1}{2}\cos\theta$	$y=2\sin\theta$
0	$\frac{1}{2}=0.5$	$0=0$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{4}\approx 0.354$	$2\sqrt{2}\approx 1.41$
$\frac{\pi}{2}$	$0=0$	$2=2$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{4}\approx -0.354$	$2\sqrt{2}\approx 1.41$
π	$-\frac{1}{2}=-0.5$	$0=0$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{4}\approx -0.354$	$-2\sqrt{2}\approx -1.41$
$\frac{3\pi}{2}$	$0=0$	$-2=-2$
$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{4}\approx 0.354$	$-2\sqrt{2}=-1.41$
2π	$\frac{1}{2}=0.5$	$0=0$



$x=\frac{1}{2}\cos\theta\ ,\ y=2\sin\theta$

$\cos\theta=2x\ ,\ \sin\theta=\frac{y}{2}$

*trig identity: $\cos^2\theta+\sin^2\theta=1$

$(2x)^2+\left(\frac{y}{2}\right)^2=1 \rightarrow \boxed{4x^2+\frac{y^2}{4}=1}$