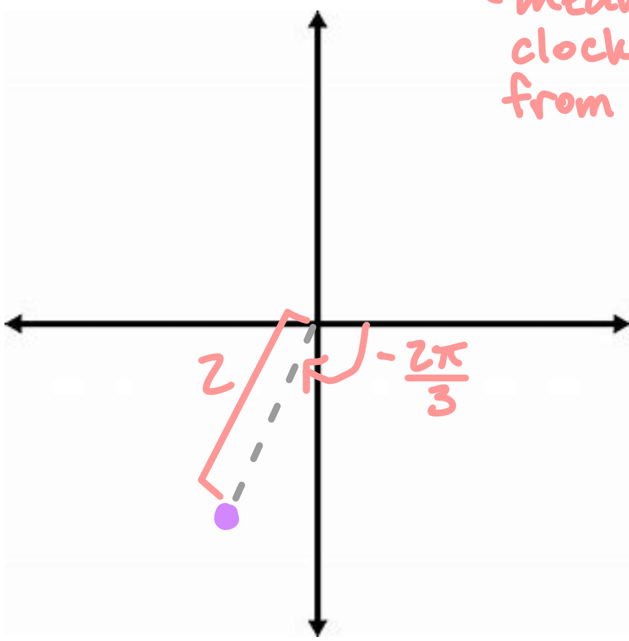
Exam 5 Review

1. Plot the polar coordinate $(2, -\frac{2\pi}{3})$.

means clockwise from x-axis!



2. Convert $\overset{(r,\theta)}{(2, \frac{\pi}{3})}$ from polar to cartesian coordinates.

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$x = 2\cos(\frac{\pi}{3})$ $\qquad$ $y = 2\sin(\frac{\pi}{3})$

$x = 2(\frac{1}{2})$ $\qquad$ $y = 2(\frac{\sqrt{3}}{2})$

$x = 1$ $\qquad$ $y = \sqrt{3}$ $\rightarrow \boxed{(1, \sqrt{3})}$

3. Convert $\overset{(x,y)}{(1, -1)}$ from cartesian to polar coordinates.

$r = \sqrt{x^2+y^2} = \sqrt{(1)^2+(-1)^2} = \sqrt{2}$

$\theta = \tan^{-1}(\frac{y}{x}) = \tan^{-1}(\frac{-1}{1}) = \tan^{-1}(-1) = \frac{7\pi}{4}$

$$\boxed{(\sqrt{2}, \frac{7\pi}{4})}$$

4. Change $y = x^2$ from cartesian to polar coordinates.

$y = r\sin\theta \qquad x = r\cos\theta$

$r\sin\theta = r^2\cos^2\theta$

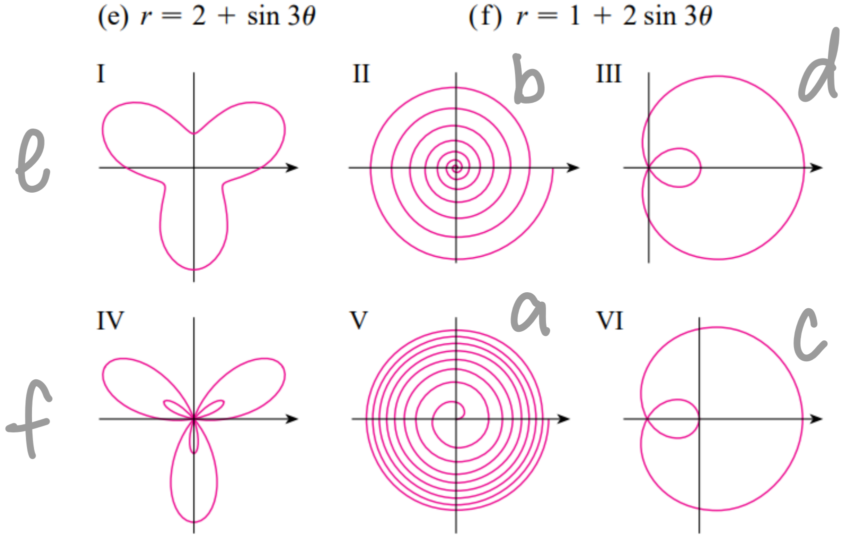
$\frac{r\sin\theta}{r\sin\theta} = \frac{r^2\cos^2\theta}{r\sin\theta}$

$1 = r \cdot \frac{\cos\theta}{\sin\theta} \cdot \cos\theta$

$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} \rightarrow \boxed{r = \tan\theta\sec\theta}$

5. Match the polar equations to their graphs.

(a) $r = \sqrt{\theta}, \quad 0 \leq \theta \leq 16\pi$
(b) $r = \theta^2, \quad 0 \leq \theta \leq 16\pi$
(c) $r = \cos(\theta/3)$
(d) $r = 1 + 2\cos\theta$
(e) $r = 2 + \sin 3\theta$
(f) $r = 1 + 2\sin 3\theta$



6. Find the derivative of the polar equation $r = 3sin(\theta)$. Then find the slope of a line tangent to r at $\theta = \frac{\pi}{3}$.

$$\frac{dy}{dx} = \frac{r' \sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$r = 3\sin\theta$, $r' = 3\cos\theta$

$$\frac{dy}{dx} = \frac{3\cos\theta \cdot \sin\theta + 3\sin\theta\cos\theta}{3\cos\theta\cos\theta - 3\sin\theta\sin\theta} = \frac{6\cos\theta\sin\theta}{3\cos^2\theta - 3\sin^2\theta}$$

$\sin(2\theta) = 2\sin\theta\cos\theta$, $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$$\frac{3\sin(2\theta)}{3\cos(2\theta)} = \boxed{\tan(2\theta)}$$

when $\theta = \frac{\pi}{3}$

$$\tan\left(\frac{2\pi}{3}\right) = \boxed{-\sqrt{3}}$$

7. Determine the equation of the line tangent to $r = 3 + 8sin(\theta)$ at $\theta = \frac{\pi}{6}$

equation of a line: $y = m(x - x_1) + y_1$

need a point (x_1, y_1) and a slope (m)

derivate = <u>slope</u> of a tangent line $\left(\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = m\right)$

<u>slope</u>: $\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$, $r = 3 + 8\sin\theta$, $r' = 8\cos\theta$

$$\frac{dy}{dx} = \frac{(8\cos\theta)(\sin\theta) + (3+8\sin\theta)(\cos\theta)}{(8\cos\theta)(\cos\theta) - (3+8\sin\theta)(\sin\theta)}$$

$$= \frac{8\cos\theta\sin\theta + (3\cos\theta + 8\cos\theta\sin\theta)}{8\cos^2\theta - (3\sin\theta + 8\sin^2\theta)} = \frac{16\cos\theta\sin\theta + 3\cos\theta}{8\cos^2\theta - 3\sin\theta - 8\sin^2\theta}$$

when $\theta = \frac{\pi}{6}$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \frac{16\cos(\frac{\pi}{6})\sin(\frac{\pi}{6}) + 3\cos(\frac{\pi}{6})}{8\cos^2(\frac{\pi}{6}) - 3\sin(\frac{\pi}{6}) - 8\sin^2(\frac{\pi}{6})}$$

$$= \frac{16(\frac{\sqrt{3}}{2})(\frac{1}{2}) + 3(\frac{\sqrt{3}}{2})}{8(\frac{\sqrt{3}}{2})^2 - 3(\frac{1}{2}) - 8(\frac{1}{2})^2} = \frac{4\sqrt{3} + \frac{3}{2}\sqrt{3}}{6 - \frac{3}{2} - 2} = \frac{\frac{11}{2}\sqrt{3}}{\frac{5}{2}} = \frac{11\sqrt{3}}{5} \rightarrow m = \frac{11\sqrt{3}}{5}$$ *

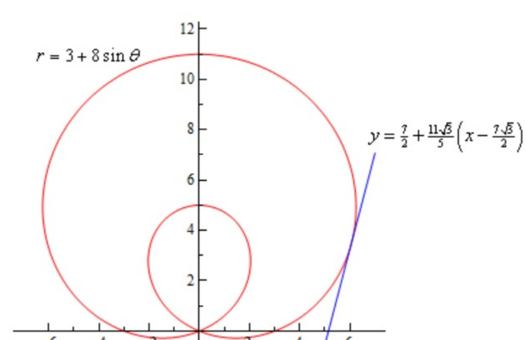
<u>point</u>: $r = 3 + 8\sin(\frac{\pi}{6}) = 3 + 8(\frac{1}{2}) = 7$. in polar $(\frac{7}{2}, \frac{\pi}{6})$

$x = r\cos\theta = 7\cos(\frac{\pi}{6}) = 7(\frac{\sqrt{3}}{2}) = \frac{7\sqrt{3}}{2}$

$y = r\sin\theta = 7\sin(\frac{\pi}{6}) = 7(\frac{1}{2}) = \frac{7}{2}$

* $(x_1, y_1) = (\frac{7\sqrt{3}}{2}, \frac{7}{2})$

<u>Visual:</u>



<u>plug into equation of a line:</u>

$$\boxed{y = \frac{11\sqrt{3}}{5}\left(x - \frac{7\sqrt{3}}{2}\right) + \frac{7}{2}}$$

8. Find the arc length of the polar equation $r =- 4sin(\theta)$ from $0 \le \theta \le \pi$.

$$\int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \,, \quad \frac{dr}{d\theta} = -4\cos\theta$$

$$\int_0^{\pi} \sqrt{(-4\sin\theta)^2 + (-4\cos\theta)^2}\, d\theta$$

$$= \int_0^{\pi} \sqrt{16\sin^2\theta + 16\cos^2\theta}\, d\theta$$

$$= \int_0^{\pi} \sqrt{16(\sin^2\theta + \cos^2\theta)}\, d\theta$$

* $\sin^2\theta + \cos^2\theta = 1$
pythag identity

$$= \int_0^{\pi} 4\, d\theta = \boxed{4\pi}$$