11.9 - Representing Functions as Power Series

1. Rewrite $f(x) = \frac{2}{1+2x}$ as a power series centered at 0 and find its interval of convergence.

$$\frac{2}{1+2x} \rightarrow \frac{2}{1-(-2x)}$$

$$\boxed{\sum_{n=0}^{\infty} 2(-2x)^n}$$

$$|-2x| < 1$$

$$-1 < -2x < 1$$

$$1 > 2x > -1$$

$$\boxed{-\frac{1}{2} < x < \frac{1}{2}}$$

2. Rewrite $f(x) = \frac{8}{2x-9}$ as a power series centered at 3 and find its interval of convergence.

$$\frac{8}{2x-9} \rightarrow 2(x-3) - -9$$

$$(2x-6)+6-9$$

$$2(x-3)-3$$

$$\frac{8}{2(x-3)-3} \quad \frac{(-1/3)}{(-1/3)} = \frac{-8/3}{1-2/3(x-3)}$$

$$\boxed{\sum_{n=0}^{\infty} -\frac{8}{3}\left(-\frac{2}{3}(x-3)\right)^n}$$

$$\left|-\frac{2}{3}(x-3)\right| < 1$$

$$-1 < -\frac{2}{3}(x-3) < 1$$

$$\frac{3}{2} > x-3 > -\frac{3}{2}$$

$$\boxed{\frac{3}{2} < x < \frac{9}{2}}$$

3. Rewrite $f(x) = \frac{5x+1}{2x^2-x-1}$ as a power series <u>centered at 0</u> and find its interval of convergence.

$$\frac{5x+1}{2x^2-x-1} = \frac{A}{2x+1} + \frac{B}{x-1}$$

$$A(x-1) + B(2x+1) = 5x+1$$

$x=1:$ $\quad 0A + 3B = 6$

$$B = 2$$

$x=-\frac{1}{2}:$ $\quad -\frac{3}{2}A + 0B = -\frac{3}{2}$

$$A = 1$$

$$\frac{1}{2x+1} + \frac{2}{x-1}\frac{(-1)}{(-1)} \rightarrow \frac{1}{1-(-2x)} + \frac{-2}{1-x}$$

$$\sum_{n=0}^{\infty} (-2x)^n + \sum_{n=0}^{\infty} -2(x)^n$$

$$-1 < x < 1$$

$$-1 < -2x < 1$$

Interval of convergence $\boxed{-\frac{1}{2} < x < \frac{1}{2}}$

$$\boxed{\sum_{n=0}^{\infty} ((-2)^n - 2)x^n}$$

$$\begin{array}{r|l} -5 & 4 \\ -1 & \\ 5 & \end{array}$$

$5x^2+5x-x-1$

$5x(x+1)-1(x+1)$

4. Rewrite $f(x) = \frac{42x-6}{5x^2+4x-1}$ as a power series centered at 0 and find its interval of convergence.

$$\frac{42x-6}{5x^2+4x-1} = \frac{A}{5x-1} + \frac{B}{x+1}$$

$$42x-6 = A(x+1) + B(5x-1)$$

$x=-1: \; -48 = 0A - 6B$

$B = 8$

$x = \frac{1}{5}: \; \frac{42}{5} - \frac{30}{5} = \frac{6}{5}A + 0B$

$12 = 6A \rightarrow A = 2$

$$\rightarrow \frac{2(-1)}{5x-1(-1)} + \frac{8}{x+1} \rightarrow \frac{-2}{1-5x} + \frac{8}{1-(-x)}$$

$$\rightarrow \sum_{n=0}^{\infty} -2(5x)^n + 8(-x)^n$$

$-1 < 5x < 1 \qquad -1 < x < 1$

IOC $= \boxed{-\frac{1}{5} < x < \frac{1}{5}}$

$$\boxed{\sum_{n=0}^{\infty} (-2(5^n) + 8(-1)^n)\, x^n}$$